acterizing this device, and the optimal renovation of an image starting from a blurred one.

This discrimination is necessary to limit the domain of hard optimization. Indeed, two kinds of problem are labeled with this name, which is not strictly defined (it is linked, in fact, to the state of the art in terms of optimization):

- Some combinatorial optimization problems, for which we do not know any *fast* exact algorithms. This is the case, particularly, for problems labeled "NP-hard", that is to say, briefly, for which an exact solution is not possible in a computing time proportional to N^n, where N is the number of unknown parameters of the problem and n is an integer.
- Some continuous-variable problems, for which we do not know any algorithms able to locate a global optimum with certainty in a finite number of computations.

Efforts have been made, separately, to solve these two kinds of "difficult" problem. In the domain of continuous optimization, there exists a vast number of classical methods labeled "global optimization methods", but these techniques are often inefficient if the objective function does not possess some particular structural properties, such as convexity. In the domain of combinatorial optimization, a vast number of *heuristics*, which produce nearly optimal solutions, have been developed; but most of these heuristics have been tailored specifically to a unique problem.

Metaheuristics

The appearance of a new kind of method, called *metaheuristics*, has allowed both of the preceding domains to become friends again: indeed, these methods can be applied to various sorts of combinatorial optimization problems and can also be adapted to continuous problems. These methods, which include the simulated annealing method, genetic algorithms, tabu search and ant colony algorithms, have been developed since 1980 with a common aim: to solve difficult optimization problems in the best way possible. They have in common, furthermore, the following characteristics:

- They are, at least partly, *stochastic*: this approach can handle the combinatorial explosion of possibilities.
- Their origins are combinatorial: they have the advantage, crucial in the continuous case, of being *direct*, which means they do not need to compute the derivatives of the objective function.
- They are inspired by *analogies*: with physics (simulated annealing, simulated diffusion, etc.), with biology (genetic algorithms, tabu search, etc.) or with ethology (ant colony and particle swarm methods, etc.).
- They are able to guide, in a particular task, another specialized search method (for example, another heuristic or a *local* exploration method).
- They share the same drawbacks: difficulties in *tuning* the parameters of the method, and the high *computation time*.

Metaheuristics are not mutually exclusive. In the current state of research, it is often impossible to predict with certainty the efficiency of a method when it is applied to a problem. The current tendency is towards the use of *hybrid methods*, which try to exploit the specific advantages of different approaches by combining them. We shall see that metaheuristics play an important role in this book, because they have contributed to the renewal of multiobjective optimization.

To close these introductory considerations on difficult mono-objective optimization, we shall briefly analyze the source of the efficiency of metaheuristics, and then outline some extensions of these methods. This study allows us to sketch a general classification of mono-objective optimization methods.

Source of efficiency of metaheuristics

For the sake of clarity, let us take a simple example of an optimization problem: the positioning of the components of an electronic device. The objective function to be minimized is the total length of the wires, and the decision variables are the positions of the components of the circuit. The form of the objective function of this problem can be sketched as in Fig. 0.1, and it depends on the "configuration": each configuration corresponds to a particular placement of components, associated with a choice of value for each of the decision variables.

When the space of possible configurations shows a complicated structure, it is difficult to locate the global minimum c_*. We explain below the failure of a "classical" iterative algorithm, before commenting on the efficient behavior of metaheuristics.

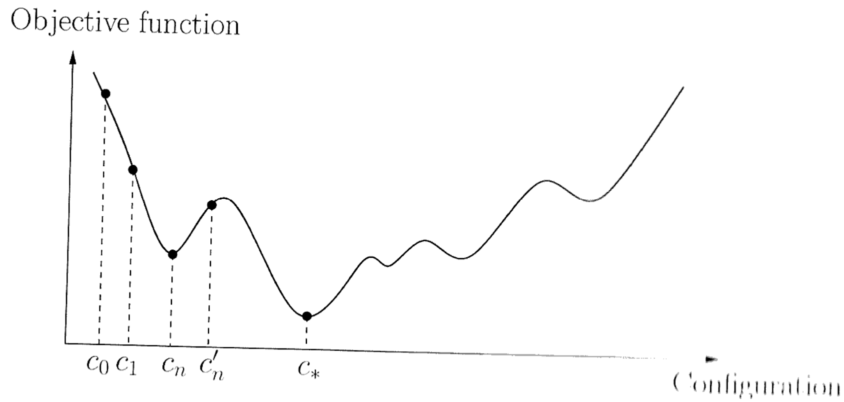

Fig. 0.1. Shape of the objective function of a difficult optimization problem depending on the "configuration".

Trapping of a "classical" iterative algorithm in a local minimum

The principle of a classical algorithm of "iterative improvement" is the following: we start with an initial configuration c_0, which can be randomly chosen, or — for example, in the case of the positioning of the components of an electronic device — be a configuration drawn by the designer. We then try to perform an elementary modification, often called a "*movement*" (for example, we permute two randomly chosen components, or we translate one of them), and we compare the values of the objective function before and after this modification. If the change brings about a decrease in the objective function, it is accepted, and the configuration obtained c_1, which is "close" to the previous configuration, will become the reference point for a